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# Portfolios in the Ibex 35 index: Alternative methods to the traditional framework, a comparative with the naive diversification in a pre- and post- crisis context

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## Abstract

In this paper, we present an analysis of the effectiveness of various portfolio optimization strategies applied to the stocks included in the Spanish Ibex 35 index, for a period of 14 years, from 2001 until 2014. The period under study includes episodes of volatility and instability in financial markets, incorporating the Global Financial Crisis and the European Sovereign Debt Crisis. This implies a challenge in portfolio optimization strategies since the methodologies are restricted to the assignment of positive weights. We have taken for asset allocation the daily returns with an estimation window equal to 1 year and we hold portfolio assets for another year. We evaluate the out-of-sample performance of 15 strategies for asset allocation in the Ibex 35 index, before and after of the Global Financial Crisis. Our results suggest that a large number of strategies outperform to the 1/N rule and to the Ibex 35 index in terms of return, Sharpe ratio and lower VaR and CVaR. The mean-variance portfolio of Markowitz with shortsale constraints, it is the only strategy that renders a Sharpe ratio statistically different to Ibex 35 index in the 2001-2007 and 2008-2014 periods

**Keywords** Portfolio optimization; Portfolio diversification; Markowitz Analysis; Naive 1/N strategy; Ibex35

**JL Classification** G11, C61.

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**Working Paper nº 1507**  
June, 2015



UNIVERSIDAD  
COMPLUTENSE  
MADRID

ISSN: 2341-2356

WEB DE LA COLECCIÓN: <http://www.ucm.es/fundamentos-analisis-economico2/documentos-de-trabajo-del-icae>

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# **Portfolios in the Ibex 35 index: Alternative methods to the traditional framework, a comparative with the naive diversification in a pre- and post- crisis context.**

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## **Abstract:**

In this paper, we present an analysis of the effectiveness of various portfolio optimization strategies applied to the stocks included in the Spanish Ibex 35 index, for a period of 14 years, from 2001 until 2014. The period under study includes episodes of volatility and instability in financial markets, incorporating the Global Financial Crisis and the European Sovereign Debt Crisis. This implies a challenge in portfolio optimization strategies since the methodologies are restricted to the assignment of positive weights. We have taken for asset allocation the daily returns with an estimation window equal to 1 year and we hold portfolio assets for another year.

This paper attempts to influence the discussion over whether the naive diversification proves to be an effective strategy as opposed to portfolio optimization models. For that, we evaluate the out-of-sample performance of 15 strategies for asset allocation in the Ibex 35 index, before and after of the Global Financial Crisis. Our results suggest that a large number of strategies outperform to the 1/N rule and to the Ibex 35 index in terms of return, Sharpe ratio and lower VaR and CVaR. The mean-variance portfolio of Markowitz with shortsale constraints, it is the only strategy that renders a Sharpe ratio statistically different to Ibex 35 index in the 2001-2007 and 2008-2014 periods.

## **1. Introduction.**

Markowitz (1952, 1959) suggested that a rational investor should choose a portfolio with the lowest risk for a given level of return instead of investing in individual assets, calling these portfolios as efficient. This approach has been the first model of portfolio selection in the literature, which is known as mean-variance of Markowitz. Although the mean-variance methodology has become the central base of the classical finance, leading directly to the development of the Capital Asset Pricing Model (CAPM) by Sharpe (1964), Lintner (1965) and Mossin (1966), the practical application is surrounded by difficulties due to their poor out-of-sample performance since the

expected returns are estimated based only on sample information, which results in an estimation error.

A latter approach to addressing the estimation error involves the application of Bayesian techniques, or shrinkage estimators. Jorion (1991) use the Bayesian approach to overcome the weakness of the expected returns estimate only by sample information. More recent approaches are based on the asset pricing model (see Shepherd, 2000; Pastor and Stambaugh, 2000); and the imposition of rules for shortselling constraint (eg, Frost and Savarino, 1988; Chopra, 1993, Jagannathan and Ma, 2003). Similarly, in the literature have been introduced the minimum-variance portfolios, based on the estimation of the covariance matrix, which is not generally as sensitive to estimation error and provides a better out of sample performance (see Chan *et al.*, 1999; Jagannathan and Ma, 2003, among others).

It is also common to use robust optimization techniques to overcome the problems of stochastic programming techniques (see, for example, Quaranta and Zaffaroni, 2008; DeMiguel *et al.*, 2009; DeMiguel and Nogales, 2009; Harris and Mazibas, 2013; Allen *et al.*, 2014A, 2014b; and Xing *et al.*, 2014). Choueifaty and Coignard (2008) and Choueifaty *et al.* (2013) proposed an approach based on the portfolio with the highest ratio of diversification. In addition, Qian (2005, 2006, 2011) introduced the portfolio with equal contribution to risk, which assigns different weights to assets so that their contribution to the overall volatility of the portfolio is proportional; the properties of this strategy were analyzed by Maillard *et al.* (2010). These methodologies aim to defend against the possible uncertainty in the parameters of the problem given that these are not exactly known.

In recent years, the interest of the authorities has increased considerably in the measurement of the effects of unexpected losses associated with extreme events in financial markets. This leads directly to improved methodologies for measurement and quantification of risk. In this sense, it is considered that the traditional framework of mean-variance, frequently used in the selection of efficient portfolios, should be revised to introduce more complex risk measures than the simple standard deviation (that is, risk measures based on the quantile). This is the context that explains the choice of Value at Risk (VaR) as synthetic risk measure that can express the market risk of a financial asset or portfolio (JP Morgan, 1994). Nevertheless, VaR has been the subject of strong criticism, despite the widespread use in banking supervision, VaR lacks subadditivity so it is not a coherent risk measure for the general distribution of loss, and this goes against the diversification principle (see Artzner *et al.*, 1997, 1999).

Moreover, the absence of convexity of VaR causes considerable difficulties in portfolio selection models based on minimizing the same. Furthermore, the VaR has been criticized for not being able to quantify the so-called “tail risk”. This has led some researchers to define new risk measures such as Conditional Value at Risk (see Rockafellar and Uryasev, 2000, 2002; Pflug, 2000; and Gaivoronski and Pflug, 2005).

There has been a rapid impulse in recent years in the literature about the use of CVaR in portfolio theory. Additionally, the CVaR has the mathematical advantage that can be minimized using linear programming methods. A simple description of the approach to minimize CVaR and CVaR constrained optimization problems can be found in Chekhlov *et al.* (2000). Krokhmal *et al.* (2002) compared the CVaR and Conditional

Drawdown-at-Risk (CDAR) approaches to minimal risk portfolios in some hedge funds. Agarwal and Naik (2004), and Giamouridis and Vrontos (2007) compared the traditional mean-variance approach with CVaR portfolios built using strategies of hedge funds.

Our objective in this paper is to compare the out of sample performance of the naive strategy regarding various models for the construction of efficient portfolios. It should be noted the existence of a debate in the literature about whether the gains from optimization are reduced by estimation errors or uncertainty in the parameters, which influence in the portfolio optimization process. In this sense, there is not consensus in the literature on whether the naive diversification shown to be more effective than other portfolio strategies (see recent works, such as DeMiguel *et al.*, 2009; You and Zhou, 2011; Kirby and Ostdiek, 2012; and Allen *et al.*, 2014th, 2014b).

For this purpose, we considered a number of optimization models: a) the classical mean-variance approach (Markowitz, 1952, 1959) and the minimum variance approach (Jagannathan and Ma, 2003); b) robust optimization techniques, as the most diversified portfolio, (see Choueifaty and Coignard, 2008; and Choueifaty *et al.*, 2013) and the equally-weighted risk contributions portfolios (see Qian, 2005, 2006, 2011); c) portfolio optimization based on Conditional Value at Risk, "CVaR" (Rockafellar and Uryasev, 2000, 2002; Alexander and Baptista, 2004; Quaranta and Zaffaroni, 2008); d) functional approach based on risk measures such as the "Maximum draw-down" (MaxDD), the "Average draw-down" (AvDD), and the "Conditional draw-down at risk" (CDAR), all proposed by Chekhlov *et al.* (2000, 2005). As well as the Conditional draw-down at risk "MinCDaR" (see Chekhlov *et al.*, 2005; and Kuutan, 2007); e) Young (1998)'s minimax optimization model, based on minimizing risk and optimizing the risk/return ratio; f) application of Copula theory to build the minimum tail-dependent portfolio, where the variance-covariance matrix is replaced by lower tail dependence coefficient (see Frahma *et al.*, 2005; Fischer and Dörflinger, 2006, and Schmidt and Stadtmüller, 2006); g) a defensive approach to systemic risk by beta strategy ("Low Beta"). The beta coefficient ( $\beta$ ) is used to assess systemic risk of an asset in the CAPM model (see Sharpe, 1964; Lintner, 1965; and Mossin, 1966), as related volatility of an asset, market, and the correlation between them. To conclude, we impose a shortselling constraint in the models.

Following DeMiguel *et al.* (2009), it is of paramount importance to compare the results of different methodologies with the "naive diversification of 1/N", which assigns equal weight to the risky assets. The 1/N strategy has proved as a difficult alternative to beat, demonstrating the practical difficulties to obtain an efficient portfolio (DeMiguel *et al.*, 2009; And Allen *et al.*, 2014a.). Therefore, we propose an efficiency analysis of the various methodologies compared with the naive diversification of 1/N and the main Spanish stock index, Ibex 35.

For the evaluation the out of sample performance, we use five criteria. The first one is the Sharpe ratio as a measure of the excess return (Sharpe, 1994). To test if the Sharpe ratio of two strategies is statistically different we obtain the p-value of the difference, using the approach suggested by Jobson and Korkie (1981), after making the correction pointed out in Memmel (2003). Similarly, we calculate the diversification ratio as a measure of the degree of portfolio diversification (Choueifaty and Coignard, 2008; and Choueifaty *et al.*, 2013.); the concentration ratio, which is simply the normalized

Herfindahl-Hirschmann index (see Hirschman, 1964); The Value at Risk (VaR) as synthetic risk measure that can express the market risk of a financial asset or portfolio, and the expected shortfall (ES or CVaR) as a coherent risk measure that takes into account the "tail risk".

As for the data, we use a sample of the daily values of the stocks included into the Ibex 35 index. The Ibex 35 index is the official index of the Spanish Continuous Market. The index is comprised of the 35 most liquid stocks traded on the Continuous market. The prices are adjusted for dividend and these are taken from Datastream. The sample period, running from January 1st 2000 to December 31st 2014, encompasses two episodes of turmoil in financial markets, such as the Global Financial Crisis, which began in 2008; and the European Sovereign Debt Crisis.

**Table 1**  
**Number of assets by time period**

Time period	N° of risky assets
03/01/2000 - 28/12/2001	21
02/01/2001 - 30/12/2002	23
02/01/2002 - 30/12/2003	25
02/01/2003 - 30/12/2004	26
02/01/2004 - 30/12/2005	29
03/01/2005 - 29/12/2006	29
02/01/2006 - 28/12/2007	30
02/01/2007 - 30/12/2008	29
02/01/2008 - 30/12/2009	30
02/01/2009 - 30/12/2010	31
04/01/2010 - 30/12/2011	31
03/01/2011 - 31/12/2012	32
02/01/2012 - 31/12/2013	33
02/01/2013 - 31/12/2014	35

We used the daily returns with an estimate window equal to one year, 252 days. Therefore, the portfolios have been built for a sample size  $N_t = 252$ , and the results have been evaluated out of sample for the next period  $N_{t+1}$ , (see Table 1). We considered only those stocks that have shown continuity within the index during the period of estimation<sup>1</sup>. We show, in Table 1, the assets number in each time period. In Appendix A (Table A2), we report the assets considered in each period.

The rest of the paper is organized as follows. In Section 2, we describe the various methodologies used for portfolio construction. In Section 3, we explain the methodology for performance evaluation. In Section 4, we show the results against the Ibex 35 index and the naive strategy of 1/N. In Section 5, we present some concluding remarks.

<sup>1</sup> In Appendix A of this paper, we include a summary table with the main statistical of the portfolios, and another table with the assets that we consider in each period.

## 2. Methodological description

### 2.1. Mean-variance portfolio

The efficient frontier of mean-variance is defined as the set of values  $(\mu_i, \sigma_i^2)$  that resolves the following multi-objective optimization problem

$$\max w^T \mu, \quad (1)$$

$$\min w^T \Sigma w,$$

$$\text{s. t. } w^T \mathbf{1} = 1,$$

where  $w^T$  is the  $(N \times 1)$  vector of weights and  $\Sigma$  denotes the variance-covariance matrix of asset returns with elements outside the diagonal and  $\sigma_{ij}$  and  $\sigma_i^2$  the  $i$ -th element of the main diagonal.

Each point on the efficient frontier  $(\mu_i, \sigma_i^2)$  corresponds to an efficient portfolio where the investor gets a maximum return for a given level of risk  $\sigma_i$ . The efficient frontier of mean-variance reflects the relationship between return and risk, introducing the tradeoff concept of risk-return in the financial markets. Therefore, it describes the level of return  $\mu_i$  given a risk exposure  $\sigma_i$ , or seen from a reverse perspective, the lower variability  $\sigma_i$  for a return level  $\mu_i$  (Markowitz, 1952, 1959).

A risk-averse rational investor will make an investment decision on the efficient frontier when the risky asset returns exhibit a multivariate normal distribution or if the utility function is quadratic. The best choice will reflect the investor's willingness to trade off risk against expected return.

To solve efficiently the problem of quadratic optimization with two objectives described above, the problem can be converted into a quadratic optimization problem for different levels of return  $\mu_i$  (Tsao, 2010).

$$\min w^T \Sigma w, \quad (2)$$

$$\text{s. t. } w^T \mu = \mu_i,$$

$$\text{s. t. } w^T \mathbf{1} = 1,$$

$$\text{s. t. } w^T \geq 0.$$

The expected return and the variance of the portfolio are  $w^T \mu$ , and  $w^T \Sigma w$ , respectively. In this paper, we solve the above quadratic optimization problem and establish an expected return  $\mu_i$  equal to the average return on the assets that are

considered in the optimization problem. We have also included a shortselling restriction such that  $w^T \geq 0$ .

## 2.2. Minimum-variance portfolio

We use the previous optimization problem to assign the weights  $w^T$  to each asset in the minimum-variance portfolio, but not including the restriction on returns,  $w^T \mu = \mu_i$ .

$$\min w^T \sum w, \quad (3)$$

$$\text{s. t. } w^T 1 = 1,$$

$$\text{s. t. } w^T \geq 0.$$

We obtain the portfolio that provides the minimum variance  $\sigma_i^2$ , given any return  $\mu_i$  in the efficient frontier of mean-variance. In contrast to the mean-variance portfolio, the minimum variance weight vector does not depend of the expected return on assets (see Jagannathan and Ma, 2003, for a study of the properties).

## 2.3. Naive diversification

Several studies confirm the existence of some investors who distribute their wealth through naive diversification strategy, they invest in a few assets alike (see Benartzi and Thaler, 2001; and Huberman and Jiang, 2006). This fact does not prove that the naive diversification is a good strategy, since investors may select a portfolio that is not within the efficient frontier, or they may choose the wrong point in it. Both situations involve a cost, where the second cost is the most important (see Brennan and Torous, 1999).

The naive strategy involves a weight distribution  $w_j = 1/N$  for all risky assets in the portfolio. This strategy ignores the data and does not involve any estimation or optimization. DeMiguel *et al.* (2009) suggest that the expected returns are proportional to total risk instead systematic risk.

## 2.4. Equal risk contributed portfolio

The portfolios built under the criterion of minimum variance and equally weighted (naive strategy 1/N) are of great interest because they are not based on the expected average returns and therefore they are supposed to be robust. Although the minimum-variance portfolios generally have the disadvantage of a high concentration ratio, it can be limited through diversification (see Qian, 2005).

Here is where the equal risk contributed (ERC) portfolios is located, which assigns different weights to assets so that the contribution of these on total portfolio volatility is proportional. Therefore, the diversification is achieved by a weight vector, which is

characterized by a distribution of less concentrated portfolio. The ERC portfolio was introduced in the literature by Qian (2005, 2006, 2011) and their properties were analyzed by Maillard *et al.* (2010).

Maillard *et al.* (2010) showed that when it comes to the standard deviation of the portfolio, the ERC solution takes an intermediate position between a minimum-variance portfolio and equally weighted portfolio. Therefore, the resulting portfolio is similar to a minimum-variance portfolio under additional diversification restrictions.

Let  $M(w_i, \dots, w_N)$  denote a measure of homogeneous risk which is a weight function  $w_i$  of each asset in the portfolio. By Euler's theorem  $M = \alpha \sum_{i=1}^N w_i \frac{\partial M}{\partial w_i}$ , where  $\alpha$  is the degree of homogeneity of  $M$ . This leads us to consider the contribution to the risk of asset  $i$  to be defined in the form

$$C_i M_{w \in \Omega} = w_i \frac{\partial M_{w \in \Omega}}{\partial w_i}. \quad (4)$$

The measure of risk  $M_{w \in \Omega}$  can be the standard deviation of the portfolio, the value at risk or the expected shortfall if the degree of homogeneity is one. The portfolio risk is equal to the sum of the risk contributions. If we introduce the formula for the standard deviation portfolio  $\sigma(w) = \sqrt{w' \Sigma w}$  to  $M_{w \in \Omega}$ , then the partial derivatives in the above equation are given by

$$\frac{\partial \sigma(w)}{\partial w_i} = \frac{w_i \sigma_i^2 + \sum_{i \neq j}^N w_i \sigma_{ij}}{\sigma(w)}. \quad (5)$$

These  $N$  partial derivatives are proportional to the  $i$ th row of  $(\Sigma w)_i$ , so the problem for the ERC portfolio with a shortsale constraints and a budget constraint is

$$\begin{aligned} P_{ERC}: w_i (\Sigma w)_i &= w_j (\Sigma w)_j, \forall i, j, \\ 0 &\leq w_i \leq 1, \\ w' \mathbf{i} &= 1, \end{aligned} \quad (6)$$

where  $\mathbf{i}$  is an  $(N \times 1)$  vector of ones. The optimal solution of ERC is valid if the value of the objective function is zero, and this only occurs when all contributions are equal risk. A closed-form solution can only be derived under the assumption that all asset pairs share the same correlation coefficient. Under this assumption, the optimal weights are determined by the ratio of the inverse volatility of the  $i$ th asset and the average of the inverse asset volatilities (see Pfaff, 2013).



## 2.5. Most diversified portfolio

Choueifaty and Coignard (2008) and Choueifaty *et al.* (2013) studied the theoretical and empirical properties of portfolios when the diversification is used as criterion. To do this, they established a measure for which the degree of diversification for a long portfolio could be evaluated. We define the diversification ratio (DR) to any portfolio  $P$  as

$$DR(P) = \frac{w' \sigma}{\sqrt{w' \Sigma w}}. \quad (7)$$

The numerator is the weighted average volatility of the individual assets, divided by the volatility of the portfolio. This relationship has a lower limit of one in the case of a portfolio composed only by an asset. Choueifaty *et al.* (2013) show that the portfolio characterized by a highly concentrated or with an asset returns very correlated would qualify as being poorly diversified, so that

$$DR(P) = \frac{1}{\sqrt{(\rho + CR) - \rho CR}}, \quad (8)$$

where  $\rho$  denotes the volatility-weighted average correlation and  $CR$  is the volatility-weighted concentration ratio. The DR only depends on the volatility-weighted average correlations in the case of a naive allocation.

Choueifaty *et al.* (2013) established the conditions for the most diversified portfolio by introducing a set of synthetic assets that share the same volatility, such that

$$D(S) = \frac{S' \Sigma_S}{\sqrt{S' V_S S}}, \quad (9)$$

where  $S$  is a portfolio composed by synthetic assets, and  $V_S$  is the covariance matrix of synthetic assets. If we have to  $S' \Sigma_S = 1$ , then to maximize  $D(S)$  is equivalent to maximize  $\frac{1}{\sqrt{S' V_S S}}$  under  $\Gamma_S$  restrictions.  $V_S$  is equal to the correlation matrix  $C$  of initial assets, so that to maximize the diversification ratio is equivalent to minimize

$$S' C S. \quad (10)$$

Thus, if the assets have the same volatility, the diversification ratio is maximized by minimizing  $w' C w$ . Therefore, the objective function coincides with the minimum-variance portfolio, although it is used the correlation matrix.

The impact of asset volatility is lower in the more diversified portfolio compared with the minimum-variance portfolio (see Pfaff, 2013). The weights are retrieved by intermediate vector rescaling weights with standard deviations of asset returns. The optimal weight vector is determined in two steps: first, an allocation is determined that yields a solution for a least correlated asset mix. This solution is then inversely adjusted by the asset volatilities, and later, the weights of the assets are adjusted inversely by their volatilities.

## 2.6. Minimum tail-dependent portfolio

Minimum tail-dependent portfolio is determined through replacing the variance-covariance matrix by the tail dependence coefficient matrix. In that sense, the lower tail of the correlation coefficient measures the dependence of the relationship between the asset returns when these are extremely negative. It is possible to find a scheme with various nonparametric estimators for minimum tail-dependent portfolio in Frahma *et al.* (2005), and Dörflinger Fischer (2006) and Schmidt and Stadtmüller (2006).

The copula theory was introduced by Sklar (1959). Sklar's theorem states that there is a  $C$  function, called copula, which establishes the functional relationship between the joint distribution and their marginal one-dimensional. Formally, let  $x = (x_1, x_2)$  be a two-dimensional random vector with joint distribution function  $F(x_1, x_2)$  and marginal distributions  $F_i(x_i), i = 1, 2$ ; there will be a copula  $C(u_1, u_2)$  such that

$$F(x_1, x_2) = P(X_1 < x_1, X_2 < x_2) = C(F_1(x_1), F_2(x_2)). \quad (11)$$

Moreover, Sklar's theorem also provides that if  $F_i$  are continuous, then the copula  $C(u_1, u_2)$  is unique. An important feature of copula is that it allows different degrees of dependency on the tail. The upper tail dependence ( $\lambda_U$ ) exists when there is a positive likelihood that positive outliers are given jointly; while the lower tail dependence  $\lambda_L$ , exists when there is a negative likelihood that negative outliers are given jointly (see Boubaker and Sghaier, 2013). Thereby, we define the lower tail dependence coefficient as follows

$$\lambda_L = \lim_{u \rightarrow 0} \frac{C(u, u)}{u}. \quad (12)$$

This limit can be interpreted as a conditional probability; therefore, the lower tail dependence coefficient is limited in the range  $[0, 1]$ . The limits are: for an independent copula ( $\lambda_L = 0$ ), and for a co-monotonic copula ( $\lambda_L = 1$ ). Nonparametric estimators for  $\lambda_L$  are derived from empirical copula.

For a given sample paired observations  $N, (X_1, Y_1), \dots, (X_N, Y_N)$ , with order statistics  $X_{(1)} \leq X_{(2)} \dots \leq X_{(N)}$  and  $Y_{(1)} \leq Y_{(2)} \dots \leq Y_{(N)}$ , the empirical copula is defined as

$$C_N\left(\frac{i}{N}, \frac{j}{N}\right) = \frac{1}{N} \sum_{l=1}^N I(X_l \leq X_{(i)} \wedge Y_l \leq Y_{(j)}), \quad (13)$$

with  $i, j = 1, \dots, N$  and  $I$  is the indicator function, which has a value of 1 if the condition in parentheses is true.  $C_N$  takes a zero value for  $i, j = 0$ .

In the literature, there are several consistent and asymptotically efficient estimators of  $\lambda_L$ , although this depend on a threshold parameter  $k$ , that is the number of statistical order. It is very important to correctly select  $k$  in order to estimate the lower tail dependence coefficient, if  $k$  is too small, this will result in an inaccurate estimation and a high bias.

For example, the following nonparametric method for estimating of  $\lambda_L$  is derived from a mixture of co-monotonous copula and independent copula. The lower tail dependence coefficient is the weight parameter between the two copula (see Pfaff, 2013). So that

$$\lambda_L(N, k) = \frac{\sum_{i=1}^k \left( C_N \left( \frac{i}{N}, \frac{i}{N} \right) - \left( \frac{i}{N} \right)^2 \right) \left( \left( \frac{i}{N} \right) - \left( \frac{i}{N} \right)^2 \right)}{\sum_{i=1}^k \left( \frac{i}{N} - \left( \frac{i}{N} \right)^2 \right)^2}. \quad (14)$$

## 2.7. CVaR portfolio

Rockafellar and Uryasev (2000) have advocated for CVaR as a useful measure of risk. Pflug (2000) showed that CVaR is a coherent risk measure with a number of attractive and desirable properties such as monotonicity, translational invariance, positive homogeneity, further CVaR satisfies subadditivity and its convex.

CVaR is proposed as a method to calculate the market risk arising as a complementary measure to VaR. CVaR is applicable to non-symmetric distributions loss, which takes into account risks beyond the VaR. Furthermore, CVaR accomplishes convexity property with what is possible to identify a global optimum point.

The upper conditional value at risk ( $CVaR^+$ ) is defined as expected losses exceed strictly the VaR; and the lower conditional value at risk ( $CVaR^-$ ) is defined as weakly losses exceeding the VaR (greater or equal losses to VaR). Thus, the conditional value at risk is equal to the weighted average VaR and  $CVaR^+$ . CVaR quantifies the excess losses of VaR and acts as an upper bound for the VaR. Therefore, portfolios with low CVaR also have a low VaR. A number of documents apply CVaR to portfolio optimization problems (see, for example, and Uryasev Rockafellar, 2000, 2002; Andersson *et al.*, 2001; Alexander and Baptista, 200; and Rockafellar *et al.*, 2006).

In terms of selection of portfolios, CVaR can be represented as a minimization problem of nonlinear programming with an objective function given as

$$\min_{w,v} \frac{1}{na} \sum_{i=1}^n \left[ \max \left( 0, v - \sum_{j=1}^m w_j r_{i,j} \right) \right], \quad (15)$$

where  $v$  is the quantile  $\alpha$  of the distribution. In the discrete case, Rockafellar and Uryasev (2000) show that its possible to convert this problem a linear programming problem by introducing auxiliary variables, so that

$$\begin{aligned} \min_{w,d,v} \frac{1}{na} \sum_{i=1}^n d_i + v, \\ \sum_{j=1}^m w_j r_{i,j} + v \geq -d_i, \forall i \in \{1, \dots, n\}, \end{aligned} \quad (16)$$

$$\sum_{j=1}^m w_j \mu_j = C,$$

$$\sum_{j=1}^m w_j = 1,$$

$$w_j \geq 0, \forall j \in \{1, \dots, n\},$$

$$d_i \geq 0, \forall i \in \{1, \dots, n\},$$

where  $v$  represents the VaR in the coverage ratio,  $\alpha$  and  $d_i$  are deviations below the VaR (see Allen *et al.*, 2014b). If the CVaR is minimized, simultaneously, the VAR also will be minimized.

## 2.8. Optimal draw-down portfolios

They are portfolio optimization problems that try to achieve weight solutions with respect to the portfolio's draw-down. This kind of optimization was proposed by Chekhlov *et al.* (2000, 2005). The task of finding optimal portfolio allocations with respect to draw-down is of considerable interest to asset managers, as its possible to avoid, somehow, large withdrawals and/or loss of revenue management.

The draw-down of a portfolio at time  $t$  is defined as the difference between the maximum uncompounded portfolio value prior to  $t$  and its value at  $t$ . Formally, denote by  $W(w, t) = y_t' w$  the uncompounded portfolio value at time  $t$ , with  $w$  the portfolio weights for the  $N$  assets included in it, and  $y_t$  is the accumulated returns. Then the draw-down,  $\mathbb{D}(w, t)$ , is defined as

$$\mathbb{D}(w, t) = \max_{0 \leq \tau \leq 1} \{W(w, \tau)\} - W(w, t). \quad (17)$$

Chekhlov *et al.* (2000) deducted three functional measures of risk: maximum draw-down (MaxDD), average draw-down (AvDD) and conditional draw-down at risk (CDaR). CDaR is dependent on the chosen confidence level  $\alpha$ . CDaR is a measure of functional risk and not a risk measure as in the case of CVaR. The limiting cases of this family of risk functions are MaxDD and AvDD.

$$CDaR(w)_\alpha = \min_{\zeta} \left\{ \zeta \frac{1}{(1-\alpha)T} \int_0^T [\mathbb{D}(w, t) - \zeta]^+ dt \right\}, \quad (18)$$

where  $\zeta$  is a threshold value for the draw-downs, so that only  $(1-\alpha)T$  observations exceed this value.

For  $\alpha \rightarrow 1$ , CDaR approaches to the maximum draw-down:  $\text{CDaR}(w)_{\alpha \rightarrow 1} = \text{MaxDD}(w) = \max_{0 \leq t \leq T} \{\mathbb{D}(w, t)\}$ . The AvDD result for  $\alpha = 0$  is  $\text{CDaR}(w)_{\alpha=0} = \text{AvDD}(w) = (1/T) \int_0^T \mathbb{D}(w, t) dt$ .

The portfolio optimization is expressed in discrete terms and the objective is defined as maximizing the annualized average return of the portfolio, (see Pfaff, 2013).

$$R(w) = \frac{1}{dC} y_T' w, \quad (19)$$

where  $d$  is the number of years in the time interval  $[0, T]$ . In short, we consider the three functional risk measures, MaxDD, AvDD and CDaR, proposed by Chekhlov *et al.* (2000, 2005). Further, we consider the minimization of CDaR.

$$P_{\text{MaxDD}} = \arg \max_{w, u} R(w) = \frac{1}{dC} y_T' w, \quad (20)$$

$$u_k - y_k' w \leq v_1 C,$$

$$u_k \geq y_k' w,$$

$$u_k \geq u_{k-1},$$

$$u_0 = 0,$$

where  $\mathbf{u}$  denotes a  $(T + 1 \times 1)$  vector of slack variables in the program formulation, in effect, the maximum portfolio values up to time period  $k$  with  $1 \leq k \leq T$ . When the portfolio is optimized with regard to limiting of the average draw-down, only the first set of inequality constraints needs to be replaced with the discrete analogue of the mean draw-down expressed in continuous time as indicated above (see Pfaff, 2013), result to

$$P_{\text{AvDD}} = \arg \max_{w, u} R(w) = \frac{1}{dC} y_T' w, \quad (21)$$

$$\frac{1}{T} \sum_{k=1}^T (u_k - y_k' w) \leq v_2 C,$$

$$u_k \geq y_k' w,$$

$$u_k \geq u_{k-1},$$

$$u_0 = 0.$$

For the CDaR linear programming problem is necessary to introduce two additional auxiliary variables, the threshold draw-down value  $\zeta$  dependent on the confidence level  $\alpha$ , and the  $(T \times 1)$  vector  $\mathbf{z}$ , representing the weak threshold exceedances; so that

$$P_{\text{CDaR}} = \arg \max_{w, u, z, \zeta} R(w) = \frac{1}{dC} y_T' w, \quad (22)$$

$$\zeta + \frac{1}{(1 - \alpha)T} \sum_{k=1}^T z_k \leq v_3 C,$$

$$z_k \geq u_k - y_k' w - \zeta,$$

$$z_k \geq 0,$$

$$u_k \geq y_k' w,$$

$$u_k \geq u_{k-1},$$

$$u_0 = 0.$$

The minimization of CDaR (see Cheklov *et al.*, 2005; and Kuutan, 2007) can be obtained similarly to the conditional value at risk (CVaR) through linear optimization, but we have to introduce auxiliary variables

$$P_{\text{MinCDaR}} = \arg \min y + \frac{1}{(1 - \alpha)T} \sum_{t=1}^T z_t, \quad (23)$$

$$z_k \geq u_k - r_p(w, t) - y,$$

$$z_k \geq 0,$$

$$u_k \geq r_p(w, t),$$

$$u_k \geq u_{k-1},$$

where  $y$  is the threshold value of the accumulative distribution function  $D(w, t)$ , and  $z_k, u_k$  are auxiliary variables.

The limitations  $u_k \geq r_p(w, t)$ , and  $u_k \geq u_{k-1}$  replace linearly the higher value of the portfolio till the moment  $t$ :  $\max\{r_p(w, t)\}$ . The first constraint ensures that  $u_k$  is always higher or at least equal to the portfolio accumulated return in the moment  $k$ , and the second constraint ensures that  $u_k$  is always higher or at least equal to the previous value (see Kuutan, 2007). Before of the optimization process,  $y$  is a free variable, after of the optimization process its the  $CDaR_\alpha$  for the MinCDaR portfolio. Thus, if we minimize the function  $H_\alpha(w, y)$ , we simultaneously obtain both values (see Albina Unger, 2014).

## 2.9. Minimum tail-dependent portfolio based in Clayton copula and low beta strategy.

The minimum tail-dependent is derived from a Clayton copula. The Clayton copula belongs to the family of Archimedean copula, its one of the most used in the literature (see Clayton, 1978). An Archimedean generator, or generator, is a continuous decreasing function  $\psi: [0, \infty] \rightarrow [0, 1]$  which complies  $\psi(0) = 1, \psi(\infty) := \lim_{t \rightarrow \infty} \psi(t) = 0$ , and that is strictly decreasing on  $[0, \inf\{t: \psi(t) = 0\}]$ . The set of all functions is denoted by  $\Psi$ .

An Archimedean generator  $\psi \in \Psi$  is called strict if  $\psi(t) < 0$  for all  $t \in [0, \infty]$ . A d-dimensional copula  $C$  is called Archimedean (see Hofert y Scherer, 2011) if it allows the representation

$$C(u) = C(u; \psi) := \psi(\psi^{-1}(u_1) + \dots + \psi^{-1}(u_d)), u \in I^d, \quad (24)$$

for some  $\psi \in \Psi$  with inverse  $\psi^{-1}: [0, 1] \rightarrow [0, \infty]$ , where  $\psi^{-1}(0) := \inf\{t: \psi(t) = 0\}$ . There are different notations for Archimedean copula. A bivariate Clayton copula can be presented so that

$$C(u_1, u_2) = \psi^{-1}(\psi(u_1) + \psi(u_2)) = (u_1^{-\delta} + u_2^{-\delta} - 1)^{1/\delta}. \quad (25)$$

The Clayton copula has the minimum tail-dependent. The coefficient is calculated according to  $\lambda_l = 2^{-1/\delta}$ . For the bivariate Clayton copula, the following simplifications are given

$$\hat{\delta} = \frac{2\hat{\rho}\tau}{1 - \hat{\rho}\tau}, \quad (26)$$

$$\hat{\theta} = \frac{1}{1 - \hat{\rho}\tau}, \quad (27)$$

where  $\hat{\rho}\tau$  is the empirical Kendall rank correlation (see, for example, Favre Genest and 2007).

In addition, we implemented the strategy of lower beta coefficient (“Low Beta”), beta ( $\beta$ ) is the coefficient used to evaluate systemic risk of an asset in the CAPM model, (see Sharpe, 1964; Lintner, 1965; and Mossin, 1966), and it relates the volatility of an asset, market, and the correlation between them.

We select assets whose volatility is less than the reference market, in absolute terms, for the construction of the beta portfolio. The process to build the portfolio can be summarized so that, we get the beta coefficients of each asset such that

$$\beta_i = \frac{Cov(R_i, R_b)}{\sigma_b^2}, \quad (28)$$

where the numerator represents the covariance between assets  $i$  and the market  $b$ , and the denominator is the variance of the market.

Then, we select those assets whose  $\beta$  coefficients and coefficients of tail dependence are below their respective medians. Finally, we get the weights by applying an inverse logarithmic scale (this application can be seen in Pfaff, 2013). Both strategies are referred to as defensive relative to the market (benchmark), as they are aimed at minimizing systemic risk.

## 2.10. Minimax portfolios based on risk minimization and optimization of the risk/return ratio

The Minimax model (see Young, 1998) aims to minimize the maximum expected loss, thus its a very conservative criterion. Formally, when it applied to the selection of portfolios, given  $N$  assets and  $t$  periods, the model can be presented as a linear programming problem, such that

$$\begin{aligned} \min_{M_p, w} M_p, & \tag{29} \\ M_p - \sum_{j=1}^m w_j r_{i,j} & \leq 0, \forall i = \{1, \dots, n\}, \\ \sum_{j=1}^m w_j \mu_j & = C, \\ \sum_{j=1}^m w_j & = 1, \\ w_j & \geq 0, \forall j \in \{1, \dots, n\}, \end{aligned}$$

where  $M_p$  is the target value to minimize, which represents the maximum loss of the portfolio given a weight vector  $w$ ,  $C$  is a certain minimum level of return, and  $\mu$  denote the forecast for the returns vector of  $m$  values. In principle, Minimax is consistent with the theory of expected utility in the limit based on a very risk adverse investor. Furthermore, the minimax model is a good approximation to the mean-variance model when the asset returns follow a multivariate normal distribution.

If we draw the portfolios set for different levels of  $C$  (using an equality rather than inequality), its possible to generate the frontier portfolio from which the optimal risk portfolio can be chosen. It is possible to estimate the optimal risk/return using fractional programming as its described in Charnes and Cooper (1962), and more recently in Stoyanov, Rachev y Fabozzi (2007). The Minimax linear programming problem can be reformulated, so that



$$\min_{M_{p,wb}} M_p, \quad (30)$$

$$\begin{aligned} M_p - \sum_{j=1}^m w_j r_{i,j} &\leq 0, \forall i = \{1, \dots, n\}, \\ \sum_{j=1}^m w_j \mu_j &= 1, \\ \sum_{j=1}^m w_j &= b, \\ b &\geq 0, \end{aligned}$$

where  $b$  is the multiplier coefficient added to the optimization problem as result of transformation of the risk/return problem. More details can be found in Charnes and Cooper (1962) for LP (Linear Programing), and in Dinkelbach (1967) for NLP (Non-linear Programing).

In summary, we use two types of optimization: the first optimization is based on risk minimization, and the second optimization is based on the risk/return ratio.

**Table 2**

**List of asset-allocation models considered**

Methodology	Model	Abbreviation
1. Naive Diversification	• Naive strategy of 1/N	1/N
2. Classic	• Mean-variance portfolio	M-V
3. Robust Portfolios	• Minimum-variance portfolio • Most diversified portfolio • Equal risk contributed portfolio • Minimum tail-dependent portfolio	GMV MDP ERC MTD
4. CVaR Portfolio	• Conditional value at risk portfolio	CVaR
5. Draw-down Portfolios	• Maximum draw-down portfolio • Average draw-down portfolio • Conditional draw-down at risk (95%) • Minimum conditional draw-down at risk (95%)	MaxDD AvDD CDaR95 MinCDaR95
6. Minimax Portfolios	• Minimax based on risk minimization • Minimax based on the risk/return ratio	R-Minimax O-Minimax
7. Defensive Portfolios	• Minimum tail-dependent with Clayton copula • Low beta portfolio	Clayton (MTD) Beta

### 3. Methodology for Evaluating Performance

We take the out of sample daily returns for one year, and we assign the weights determined by the portfolio optimization process to each asset  $i$ . We consider five measures for statistical comparison between the portfolio strategies<sup>2</sup>: Value at Risk (VaR), Conditional Value at Risk (CVaR), Sharpe ratio, diversification ratio and concentration ratio. Results are provided for three time periods, 2001-2014, 2001-2007 and 2008-2014.

#### 3.1. Value at risk and conditional value at risk

Value at Risk (VaR) is a measure of synthetic risk that can express the market risk of a financial asset or portfolio. In general terms, VaR is the maximum potential loss that a financial asset may suffer with a certain probability for a certain period of tenure. JP Morgan tried to establish a market standard by RiskMetrics in 1994 (JP Morgan, 1994).

For a confidence level  $\alpha \in (0,1)$ , VaR is defined as the smallest number  $l$  such that the probability of loss  $L$  is not greater than  $1 - \alpha$  for greater losses than  $l$ . This value corresponds to the quantiles of loss distribution, and it can be formally expressed as

$$VaR_\alpha = \inf\{l \in \mathbb{R}: P(L > l) \leq 1 - \alpha\} = \inf\{l \in \mathbb{R}: F_L(l) \geq \alpha\}, \quad (31)$$

where  $F_L$  is the distribution function of the losses (see Pfaff, 2013).

The expected shortfall risk measure (ES or CVaR) arises due to deficiencies that VaR shows. CVaR was introduced by Artzner *et al.* (1997, 1999); Rockafellar and Uryasev (2002) showed that CVaR is a consistent measure of risk and may also take into consideration the "tail risk".

CVaR is defined for a type I error  $\alpha$  as

$$ES_\alpha = \frac{1}{1 - \alpha} \int_\alpha^1 q_u(F_L) du, \quad (32)$$

where  $q_u(F_L)$  is the quantile function of loss distribution  $F_L$ . Therefore ES can be expressed in VaR terms such that

$$ES_\alpha = \frac{1}{1 - \alpha} \int_\alpha^1 VaR_u(L) du, \quad (33)$$

ES can be interpreted as the VaR average in the range  $(1 - \alpha, 1)$ .

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<sup>2</sup> We also include the total return and the annualized return of each strategy  $z$ .

Total Return =  $\left(\frac{VF_z - VI_z}{VI_z}\right) \times 100\%$ ; Annual Return =  $\left[(VF_z / VI_z)^{1/N} - 1\right] \times 100\%$ .

### 3.2. Sharpe ratio

We calculate the out of sample annualized Sharpe ratio for each strategy  $z$ . Sharpe ratio is defined as the sample mean of out-of-sample excess returns over the risk-free asset  $\hat{\mu}_z$ , divided by their sample standard deviation  $\hat{\sigma}_z$ , such that

$$\text{Sharpe}.R = \frac{\hat{\mu}_z}{\hat{\sigma}_z}. \quad (34)$$

To test the statistical independence of the Sharpe ratios for each strategy with respect to benchmark, we calculate the p-value of the difference, using the approach suggested by Jobson and Korkie (1981) after making the correction pointed out in Memmel (2003), and recently applied in DeMiguel *et al.* (2009). So that, given two portfolios  $a$  and  $b$ , with mean  $\hat{\mu}_a$ ,  $\hat{\mu}_b$ , variance  $\hat{\sigma}_a$ ,  $\hat{\sigma}_b$ , and covariance  $\hat{\sigma}_{a,b}$  about a sample of size  $N$ , its checked by the test statistic  $\hat{z}_{JK}$ , the null hypothesis that  $H_0: \hat{\mu}_a/\hat{\sigma}_a - \hat{\mu}_b/\hat{\sigma}_b = 0$ . This test is based on the assumption that income is distributed independently and identically (IID) in time following a normal distribution, (see Jobson and Korkie, 1981; and Memmel, 2003).

### 3.3 Diversification and concentration ratios.

We define diversification ratio (DR) to any portfolio  $P$  as follows

$$DR(P) = \frac{w' \sigma}{\sqrt{w' \Sigma w}}. \quad (7)$$

The numerator is the weighted average volatility of the single assets, divided by the portfolio volatility (portfolio standard deviation). From the above equation is derived the following expression, such that

$$DR(P) = \frac{1}{\sqrt{(\rho + CR) - \rho CR}}, \quad (8)$$

where  $\rho$  denotes the volatility-weighted average correlation and CR is the volatility-weighted concentration ratio. The parameter  $\rho$  is defined as

$$\rho = \frac{\sum_{i \neq j}^N (w_i \sigma_i w_j \sigma_j) \rho_{ij}}{\sum_{i \neq j}^N (w_i \sigma_i w_j \sigma_j)}. \quad (35)$$

The concentration ratio (CR) is the normalized Herfindahl–Hirschmann index (see Hirschman, 1964)

$$CR(P) = \frac{\sum_{i=1}^N (w_i \sigma_i)^2}{\sum_{i=1}^N (w_i \sigma_i)^2}. \quad (36)$$

## 4. Results

In this section, we compare the out-of-sample results obtained for the various portfolio strategies. For that, we show the results of the five measures for statistical comparison between the portfolio strategies, contained in the previous section. The portfolio strategies results are compared with the Ibex 35 index and the naive strategy of  $1/N$ .

We take the out-of-sample daily returns for one year, and we assign the weights determined by the portfolio optimization process to each asset  $i$  considered, so that we build the portfolio and analyze it for next year. Therefore, we build portfolios with the daily returns series of  $N_t$  period and they are tested for the following period,  $N_{t+1}$ , for  $t = 2000, 2001, \dots, 2013$ . We have built 14 portfolios for methodological framework, although the results are aggregated by time periods: 2001-2014, 2001-2007 and 2008-2014.

In the first and second columns of Tables 3, 4 and 5, we present the total return (Total Return) and the annualized return (Annual Return) of each strategy for the time periods 2001-2014 (Table 3), 2001-2007 (Table 4) and 2008-2014 (Table 5). The value at risk and the conditional value at risk (1 day) appear in the third and fourth columns, respectively. The Sharpe ratio and the  $p$ -value of each strategy, including the Ibex 35 index, are shown in the fifth column. We also include the  $p$ -value of the difference for each strategy with respect to Ibex 35 index. In the last two columns, six and seven, we report the diversification and concentration ratios, respectively.

### 4.1. Out-of-sample performance: 2001-2014

Five strategies have an annual return equal or greater than 9%, compared with the Ibex 35 index, that does not exceed 1% by year. This can be seen more intuitive when considering the total return since 2001. The MinCDaR95 portfolio achieved a total return equal to 291.71%, followed by the MV, O-Minimax, Beta and GMV portfolios, with a total return greater than 240%. During the same period, the Ibex 35 index increased 13.21%, being followed in terms of lower returns by two portfolios based on the naive diversification, the  $1/N$  and ERC strategies, with a total return of 46.80% and 81.18%, respectively. All strategies have a lower VaR and CVaR than the Ibex 35 index (2.52 and 3.17), except the AvDD portfolio. The GMV portfolio stands out as the portfolio with lower VaR and CVaR (1.77 and 2.12, respectively).

Four strategies achieve an annualized Sharpe ratio of 0.5. The MV portfolio emerges with a Sharpe ratio equal to 0.566, followed by the GMV, Beta and MinCDaR95 portfolios with 0.554, 0.551 and 0.537, respectively. Considering the  $p$ -value, the above mentioned strategies turn out to be moderate or very significant, that is, their Sharpe ratios do differ statistically with respect to the Ibex 35 index. The  $1/N$  and ERC strategies render Sharpe ratio well below to the MV and GMV portfolios, indeed, if we exclude the Ibex 35 index, the  $1/N$  and ERC portfolios have the lowest Sharpe ratios.

The MDP, Clayton (MTD) and Beta strategies present the highest ratios of diversification, the first one standing out with a ratio of 1.71. The MDP portfolio is among the strategies with a higher Sharpe ratio, showing the possibility to obtain a high Sharpe ratios and the same time a considerable diversification ratio. In addition, four strategies exceed the diversification ratio of the  $1/N$  strategy, such as the MTD,

ERC, GMV and M-V portfolios. All these with a diversification ratio between the values of 1.71 (MDP portfolio) and 1.59 (MV portfolio).

**Table 3**

**Summary of main results, 2001-2014 period**

Portfolio	Total Return	Annual Return	VaR 95% 1 day	CVaR 95% 1 day	Annualized Sharpe ratio ( $p$ -value)	Diversification ratio	Concentration ratio
Ibex 35	13.21%	0.89%	2.525	3.170	0.0365 (1.000)	---	---
1/N	46.80%	2.78%	2.165	2.720	0.1321 (0.493)	1.5648	<b>0.0400</b>
M-V	<b>276.37%</b>	<b>9.93%</b>	<b>1.778</b>	<b>2.240</b>	<b>0.5663</b> (0.005)***	1.5989	0.1551
GMV	<b>244.31%</b>	<b>9.23%</b>	<b>1.689</b>	<b>2.129</b>	<b>0.5544</b> (0.009)***	1.6102	0.1513
MDP	179.21%	7.61%	<b>1.742</b>	<b>2.194</b>	<b>0.4440</b> (0.038)**	<b>1.7153</b>	0.1113
ERC	81.18%	4.34%	1.988	2.499	0.2239 (0.182)	<b>1.6164</b>	<b>0.0391</b>
MTD	117.47%	5.71%	1.836	2.310	0.3176 (0.092)*	<b>1.6326</b>	<b>0.0973</b>
CVaR	154.00%	6.88%	<b>1.786</b>	<b>2.248</b>	0.3896 (0.096)*	1.5100	0.2059
MaxDD	159.24%	7.04%	2.383	2.997	0.3041 (0.261)	1.3464	0.3830
AvDD	193.39%	7.99%	3.170	3.988	0.2585 (0.338)	1.0788	0.8401
CDaR95	167.64%	7.29%	2.437	3.066	0.3067 (0.251)	1.2765	0.4879
MinCDaR95	<b>291.71%</b>	<b>10.24%</b>	1.937	2.441	<b>0.5375</b> (0.030)**	1.4157	0.2901
R-Minimax	105.82%	5.29%	2.006	2.523	0.2707 (0.239)	1.4725	0.2141
O-Minimax	<b>247.09%</b>	<b>9.3%</b>	2.188	2.754	0.4317 (0.096)*	1.2563	0.5223
Clayton (MTD)	184.78%	7.76%	1.852	2.332	0.4257 (0.041)**	<b>1.6932</b>	<b>0.0954</b>
Beta	<b>246.43%</b>	<b>9.28%</b>	<b>1.705</b>	<b>2.148</b>	<b>0.5515</b> (0.008)***	<b>1.6783</b>	<b>0.1008</b>

Results for the period comprises between 2001 and 2014. In parenthesis, the  $p$ -value corresponding to the  $\hat{z}_{JK}$  test. The asterisks show the significance of the tests: weak significance (\*), moderate significance (\*\*), strong significance (\*\*\*). Bold values indicate the five best-performing portfolios according to each metric.

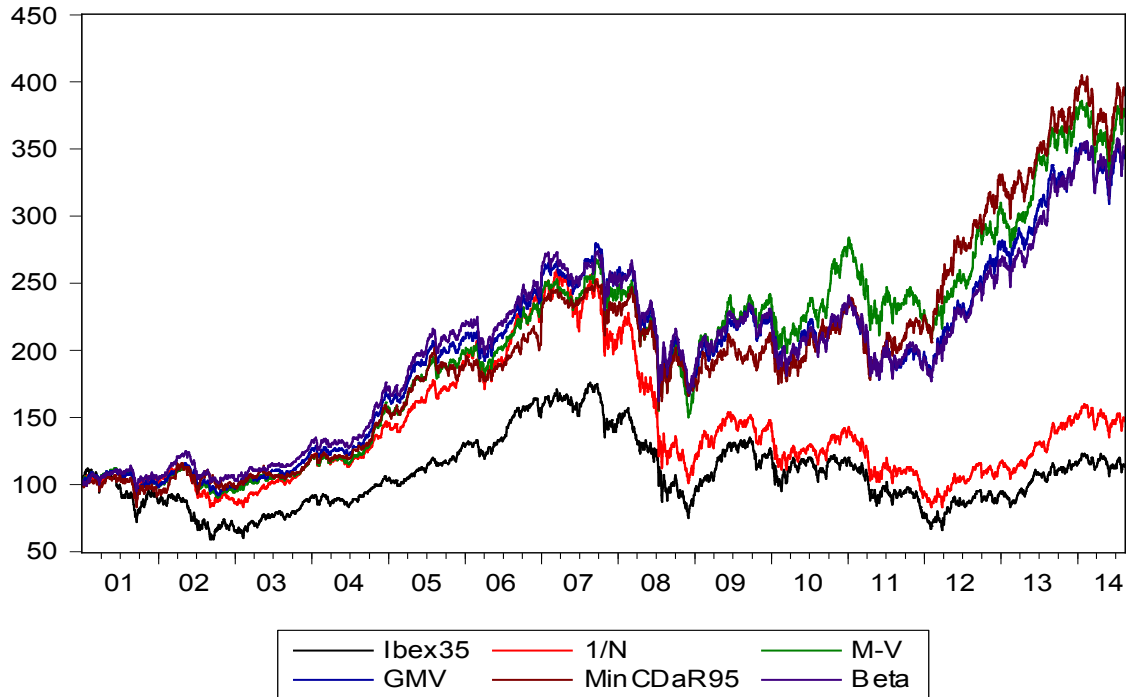
The concentration ratio rewards the largest share of assets in the portfolio, so the portfolios based on the naive diversification have the lowest concentration ratios (the ERC portfolio with 0.039 and the 1/N portfolio with 0.04), followed by two portfolios based on the lower tail dependence: the Clayton (MTD) and MTD portfolios, with 0.095 and 0.097, respectively. The concentration ratio can be related to the cost of building the portfolio because the concentration ratio decreases when the number of assets increases in the portfolio.

In Figure 1, we show the poor performance of the Ibex 35 index and the naive strategy of 1/N with respect to the other four methodologies considered (with a Sharpe ratio greater than 0.5). The differences between the Ibex 35 index and the strategies are relevant from 2002, although the greatest divergence is reached in 2014. At the end of the time period under study (the year 2014), the Ibex 35 index registered a total return of 13.21% in contrast to the rest of strategies, which achieved a minimum total return of 240%, except for the naive strategy of 1/N (with a total return of 46.80%).

The accumulated wealth generated by the naive strategy of 1/N was similar to other portfolios during the 2001-2007 period. However, the naive strategy performance is very similar to that of the Ibex 35 index in the 2008-2014 period. In short, this fact causes that the return of the 1/N portfolio in 2001-2014 period is 46.8%, clearly surpassed at least by nine strategies, among which the MinCDaR95 and the M-V portfolios stand out.

**Figure 1**

**Accumulated wealth, 2001-2014 period**



Base 100 in January 2<sup>st</sup> 2001. We represent the accumulated wealth of an investor who invested 100 currency units on January 2<sup>st</sup> 2001. We include the Ibex 35 index, the 1/N portfolio, the M-V portfolio, the GMV portfolio, the MinCDaR95 portfolio and the Beta portfolio.

Given the controversial behavior of the naive strategy from the Global Financial Crisis (2008), we need to analyze the behavior of the various strategies before and after 2008 due to the controversial behavior of the naive strategy from the Global Financial Crisis (2008). Thus, we examine separately the out-of-sample performance for the 2001-2007 and 2008-2014 time periods.

#### 4.2. Out-of-sample performance: 2001-2007

The GMV, Clayton (MTD) and Beta portfolios achieve an annual return equal or greater than 15%. In total, there are six strategies that outperform the naive strategy of 1/N, whose annual return is 13.26%. In line with the results for the entire sample period (2001-2014), the return of the Ibex 35 index is exceeded by all portfolio strategies. Thus, the MV, GMV, Clayton (MTD) and Beta portfolios get a total return equal or greater than 160%. The GMV portfolio achieves the higher performance with a total return of 173.18%, in contrast with the Ibex 35 index, with a total return of 67.20%.

Except for the AvDD portfolio, the rest of strategies dominate the Ibex 35 index (VaR of 2.03 and CVaR of 2.566) in terms of Value at Risk and Conditional Value at Risk at one day. The GMV portfolio is the strategy with the lowest VaR and CVaR (1.24 and 1.64, respectively) followed for the Beta, M-V and CVaR portfolios. In advance, one

would expect that the CVaR portfolio would obtain a lower VaR and CvaR than other portfolios; however, this strategy is overcome by the three portfolios listed previously. In addition, there are nine portfolios that show a smaller VaR and CVaR than the strategy of 1/N (1.57 and 1.99, respectively).

**Table 4**

**Summary of main results, 2001-2007 period**

Portfolios	Total Return	Annual Return	VaR 95% 1 day	CVaR 95% 1 day	Annualized Sharpe ratio ( <i>p</i> -value)	Diversification ratio	Concentration ratio
Ibex 35	67.2%	7.62%	2.038	2.566	0.3805 (1.000)	---	---
1/N	139.01%	13.26%	1.579	1.994	0.8410 (0.045)**	1.7043	<b>0.0416</b>
M-V	<b>160.57%</b>	<b>14.64%</b>	<b>1.302</b>	<b>1.648</b>	<b>1.1173</b> (0.015)**	1.7405	0.1131
GMV	<b>173.18%</b>	<b>15.44%</b>	<b>1.247</b>	<b>1.580</b>	<b>1.2237</b> (0.008)**	<b>1.7656</b>	0.1061
MDP	126.14%	12.36%	<b>1.333</b>	<b>1.684</b>	0.9267 (0.095)*	<b>1.8935</b>	<b>0.0904</b>
ERC	143.36%	13.55%	1.430	1.807	0.9458 (0.024)**	<b>1.7764</b>	<b>0.0419</b>
MTD	129.88%	12.59%	1.359	1.718	0.9280 (0.041)**	1.7629	<b>0.0794</b>
CVaR	138.64%	13.23%	<b>1.326</b>	<b>1.676</b>	<b>0.994</b> (0.074)*	1.6635	0.1581
MaxDD	124.13%	12.22%	1.708	2.154	0.7201 (0.448)	1.4408	0.3433
AvDD	84.53%	9.15%	2.795	3.518	0.3332 (0.978)	1.1311	0.7649
CDaR95	133.05%	12.85%	1.783	2.250	0.7248 (0.418)	1.4006	0.3882
MinCDaR95	<b>146.73%</b>	<b>13.77%</b>	1.623	2.049	0.8500 (0.216)	1.5151	0.2409
R-Minimax	127.53%	12.46%	1.461	1.846	0.8541 (0.127)	1.5664	0.2012
O-Minimax	67.69%	7.66%	1.882	2.369	0.4142 (0.936)	1.3620	0.4025
Clayton (MTD)	<b>168.32%</b>	<b>15.14%</b>	1.381	1.747	<b>1.0895</b> (0.051)*	<b>1.8634</b>	0.0989
Beta	<b>167.13%</b>	<b>15.07%</b>	<b>1.249</b>	<b>1.581</b>	<b>1.1946</b> (0.020)**	<b>1.8572</b>	<b>0.0928</b>

Results for the period comprises between 2001 and 2007. In parenthesis, the *p*-value corresponding to the  $\hat{z}_{JK}$  test. The asterisks show the significance of the tests: weak significance (\*), moderate significance (\*\*), strong significance (\*\*\*). Bold values indicate the five best-performing portfolios according to each metric.

The GMV, M-V, Beta, Clayton (MTD) and CvaR portfolios achieve an annualized Sharpe ratio near or above of 1, in contrast to the Sharpe ratio obtained for the Ibex 35 index (0.380). The GMV portfolio stands out with a Sharpe ratio equal to 1.223, followed by the Beta, M-V and Clayton (MTD) portfolios with 1.194, 1.117 and 1.089, respectively. In addition, there are nine portfolios whose Sharpe ratios differ statistically to the Ibex 35, this is, all portfolio strategies except those based on the minimax model and the conditional drawdown-at-risk approaches.

The MDP, Clayton (MTD) and Beta strategies have the highest diversification ratios, greater than 1.85. On the other hand, the AvDD, O-Minimax and CDaR95 strategies have low diversification ratios, none greater than 1.4. The GMV and M-V portfolios exceed the diversification ratio of the 1/N strategy. Finally, seven portfolios are able to overcome to the strategy of 1/N in terms of diversification ratio.

Again, portfolios based on the naive diversification are those that have a lower concentration ratio, slightly lower for the 1/N portfolio (0.041). In contrast, the AvDD, O-Minimax, CDaR95 and MaxDD strategies are highly concentrated, in all cases, with a concentration ratio greater than 0.34, and particularly in the AvDD portfolio with a ratio of 0.76.

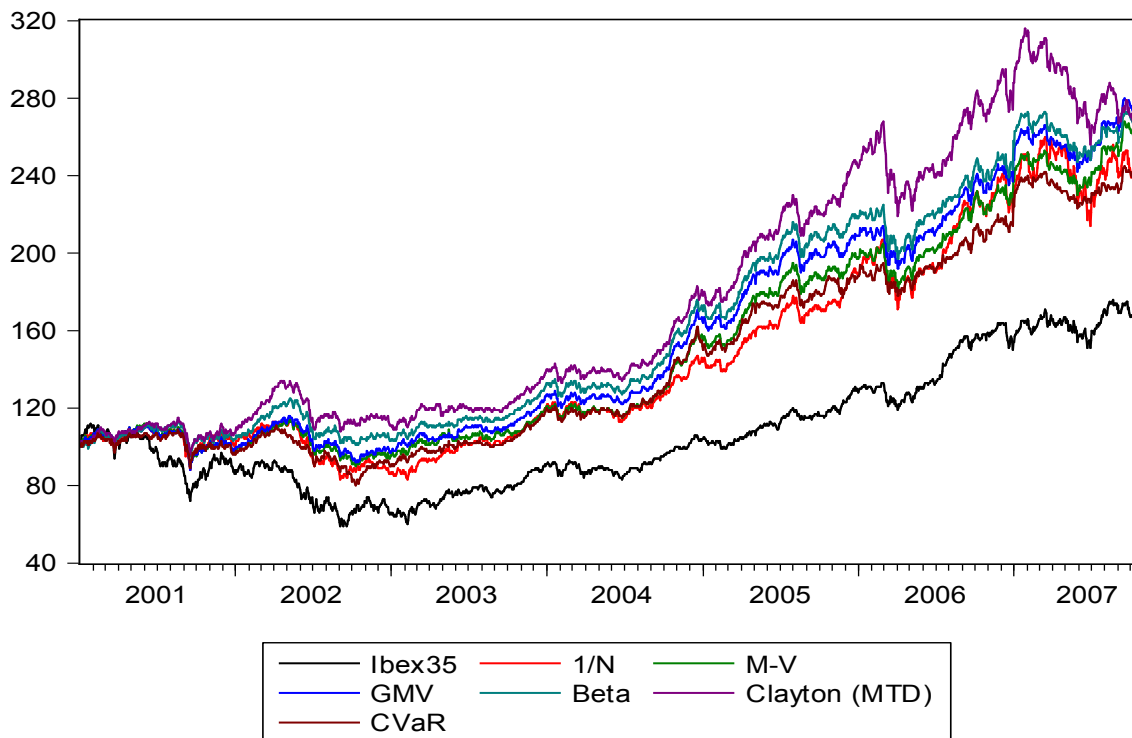
In Figure 2, we show the poor performance of the Ibex 35 index compared to the other six methodologies under evaluation (the five portfolios with higher Sharpe ratio and the

naive strategy of 1/N). As can be seen, the differences between the Ibex 35 index and the portfolios began from the middle of 2001. From 2001 to 2007, the Ibex35 index achieved a total return of just over 67%. Meanwhile, the GMV and MV strategies had a total return greater than 160%. Even the strategy of 1/N obtained double return (139.01%) than the Ibex 35 index.

The strategy of 1/N provides a good out-of-sample performance, especially when its compared with the Ibex 35 index. However, the 1/N portfolio is clearly exceeded by other strategies, not only on return but also on a higher Sharpe ratio, a lower VaR and CVaR, and greater diversification ratio. In short, there are five portfolios that completely dominate, except in concentration ratio, the naive strategy of 1/N, among which the GMV, MV and Beta portfolios stand out.

**Figure 2**

**Accumulated wealth, 2001-2007 period**



Base 100 in January 2<sup>st</sup> 2001. We represent the accumulated wealth of an investor who invested 100 currency units on January 2<sup>st</sup> 2001. We include the Ibex 35 index, the 1/N portfolio, the M-V portfolio, the GMV portfolio, the Beta portfolio, the Clayton (MTD) portfolio and the CVaR portfolio.

In conclusion, the weak out-of-sample performance of the 1/N strategy in the 2001-2014 period contrasts with the good performance of this portfolio in the 2001-2007 period; this behavior suggests that the 1/N strategy has been quite poor during the Global Financial Crisis and European Sovereign Debt Crisis.



### 4.3. Out-of-sample performance: 2008-2014

Four portfolios achieve an annual return higher of 5% in the 2008-2014 period, providing the O-Minimax portfolio the greatest return, with an annual return of around 11%. It is an exceptional case since the rest of strategies are unable to overcome such threshold of 5% by year. The return obtained is well below that achieved in the previous period, where three portfolios rendered annualized returns above 15%. The Ibex 35 index and the 1/N portfolio are in the opposite direction, with an annual return drop of 5.26% and 6.73%, respectively. Taking this into consideration, the relative performance of other strategies is not as poor like a priori might seem.

If we consider the total return for the period, the O-Minimax portfolio obtains a return of 100%, followed for the AVDD portfolio with a 58.99%, the MinCDaR95 portfolio with 58.76% and the MV portfolio with a 44.44% of total return. Meanwhile, on the opposite side the 1/N portfolio stands out with a total return of -38.58% and the Ibex 35 index with a total return of -31.48%. So the strategy of 1/N obtained negative returns even higher than those obtained by the Ibex 35 index.

**Table 5**

#### Summary of main results, 2008-2014 period

Portfolios	Total Return	Annual Return	VaR 95% 1 day	CVaR 95% 1 day	Annualized Sharpe ratio ( <i>p</i> -value)	Diversification ratio	Concentration ratio
Ibex 35	-31.48%	-5.26%	2.879	3.609	-0.1873 (1.000)	---	---
1/N	-38.58%	-6.73%	2.599	3.256	-0.2663 (0.481)	1.4252	<b>0.0383</b>
M-V	<b>44.44%</b>	<b>5.39%</b>	<b>2.126</b>	<b>2.674</b>	<b>0.2559</b> (0.065)*	<b>1.4573</b>	0.1971
GMV	26.03%	3.36%	<b>2.017</b>	<b>2.534</b>	0.1687 (0.171)	1.4548	0.1964
MDP	23.47%	3.06%	<b>2.054</b>	<b>2.580</b>	0.1508 (0.171)	<b>1.5370</b>	0.1322
ERC	-25.55%	-4.13%	2.398	3.006	-0.1765 (0.938)	1.4564	<b>0.0362</b>
MTD	-5.40%	-0.79%	2.192	2.751	-0.0368 (0.508)	<b>1.5023</b>	<b>0.1153</b>
CVaR	6.43%	0.89%	<b>2.154</b>	<b>2.705</b>	0.0422 (0.407)	1.3565	0.2538
MaxDD	15.66%	2.10%	2.846	3.575	0.0749 (0.335)	1.2521	0.4227
AvDD	<b>58.99%</b>	<b>6.85%</b>	3.436	4.321	<b>0.2010</b> (0.201)	1.0264	0.9153
CDaR95	14.84%	2.00%	2.900	3.643	0.0699 (0.357)	1.1524	0.5876
MinCDaR95	<b>58.76%</b>	<b>6.83%</b>	2.171	2.731	<b>0.3165</b> (0.081)*	1.3162	0.3392
R-Minimax	-9.54%	-1.42%	2.401	3.012	-0.0605 (0.637)	1.3786	0.2270
O-Minimax	<b>106.98%</b>	<b>10.95%</b>	2.424	3.053	<b>0.4522</b> (0.049)**	1.1506	0.6421
Clayton (MTD)	6.13%	0.85%	2.213	2.778	0.0393 (0.292)	<b>1.5229</b>	<b>0.0919</b>
Beta	<b>29.69%</b>	<b>3.78%</b>	<b>2.045</b>	<b>2.570</b>	<b>0.1872</b> (0.104)	<b>1.4993</b>	<b>0.1089</b>

Results for the period comprises between 2008 and 2014. In parenthesis, the *p*-value corresponding to the  $\hat{z}_{JK}$  test. The asterisks show the significance of the tests: weak significance (\*), moderate significance (\*\*), strong significance (\*\*\*). Bold values indicate the five best-performing portfolios according to each metric.

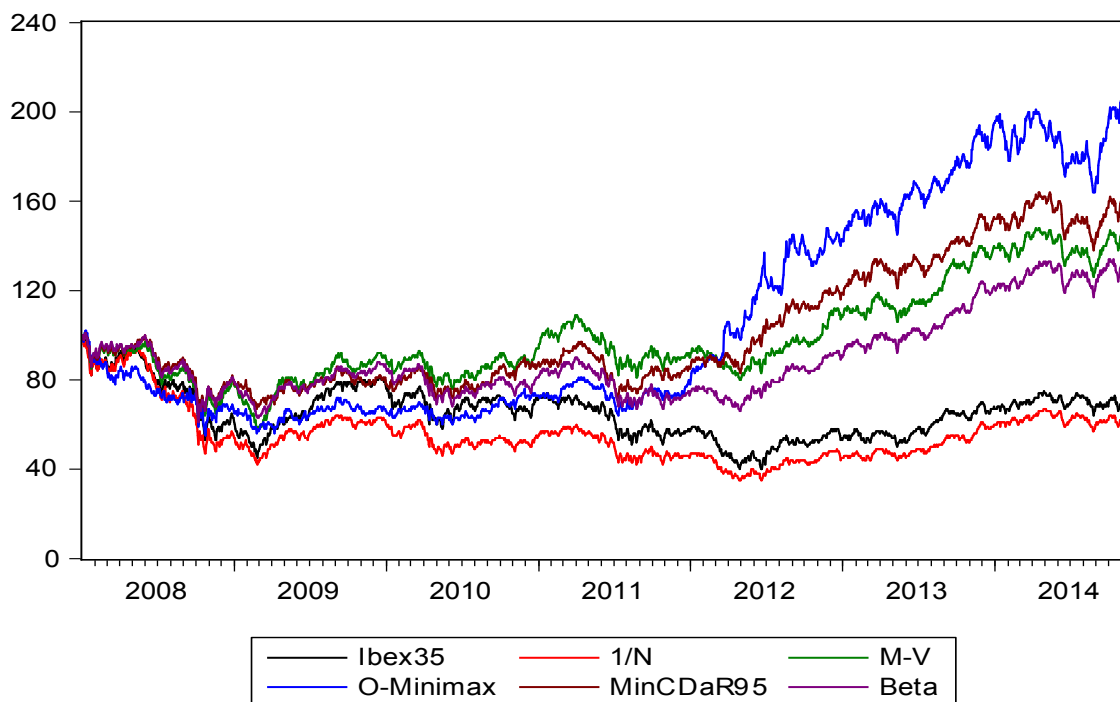
Regarding the Value at Risk and Conditional Value at Risk associate with each strategy, it is again the GMV portfolio that has a lower VaR and CVaR with 2.01 and 2.53, respectively. The GMV portfolio is followed by the Beta, MDP, MV and CVaR portfolios, in no case, with a VaR and CVaR higher than 2.2 and 2.8. These are good results if we compare them with the Ibex 35 index (2.879 and 3.609) and the 1/N strategy (2.599 and 3.256). In this regard, 11 out of the 14 portfolios have a lower VaR and CVaR with respect to the Ibex 35 index and the 1/N strategy.

All strategies, except the 1/N portfolio (-0.0266), obtained Sharpe ratios higher than that for the Ibex 35 index. However, they are only three strategies that statistically exceed the Sharpe ratio of the Ibex 35 index; this is because the covariance between the portfolio and the index is very high. The O-Minimax portfolio has the highest Sharpe ratio (0.452), and the difference from the Ibex 35 index is moderately significant. Regarding the other two portfolios: the MinCDaR95 (0.315) and the MV (0.255) portfolios, both present a relatively high Sharpe ratio, although in both cases the difference is weakly significant.

The MDP, Clayton (MTD) and MTD strategies have the highest diversification ratios, the MDP, with a remarkable ratio of 1.53, nevertheless somewhat lower than the value of the 2001-2007 period, highlighting the highest correlation between asset returns in the portfolio during the 2008-2014 period. Again, the AvDD, CDaR95 and MaxDD portfolios have the lowest diversification ratio. In total, there are seven strategies that exceed the diversification ratio of the 1/N portfolio (1.42), including the Beta (1.499), MV (1.457) and GMV (1.454) portfolios.

**Figure 3**

**Accumulated wealth, 2008-2014 period**



Base 100 in January 1<sup>st</sup> 2008. We represent the accumulated wealth of an investor who invested 100 currency unit on January 1<sup>st</sup> 2008. We include the Ibex 35 index, the 1/N portfolio, the M-V portfolio, the O-Minimax portfolio, the MinCDaR95 portfolio, and the Beta portfolio.

The ERC and 1/N portfolios have the lowest concentration ratios, with 0.036 and 0.038, respectively. The concentration ratio is slightly lower than in 2001-2007 period due to an increase in the assets number (see Table 1). In contrast, the AVDD, O-Minimax, and

CDaR95 strategies present a higher concentration ratio, in all cases with a concentration ratio greater than 0.58. This increase can be explained by the higher correlation between asset returns. This fact is widely investigated in recent papers as Moldovan (2011), for the New York, London and Tokyo index; and in Ahmad *et al.* (2013), for the contagion between financial markets.

In Figure 3, we show the poor performance of the Ibex 35 index and the 1/N portfolio compared to the other four methodologies under scrutiny (three portfolios with Sharpe ratio significantly different to the Ibex 35 index and the Beta portfolio, which are almost significant).

The differences between the 1/N portfolio and the rest of portfolios began from 2008. The 1/N portfolio performance is worse than the Ibex 35 index, in terms of total return (-38.58%). Meanwhile and during this period, the Ibex 35 index performance has been quite poor, with a total return of -31.48% and annualized Sharpe ratio of -0.187; in contrast with the performance of the O-Minimax, MinCDaR95, MV and Beta portfolios, the O-Minimax standing out with a total return of 106.98% and annualized Sharpe ratio of 0.452.

The 1/N portfolio performance before and during the Global Financial Crisis and European sovereign debt crisis indicates that this strategy has a good behavior when the market trend is bullish and *vice versa* when its bearish. The increase in the correlation between assets has adversely affected the 1/N portfolio performance (during the period 2008-2014).

## 5. Concluding remarks

In this paper, we have examined fifteen asset allocation models in the main Spanish stock market (using the Ibex 35 index). We have compared the total returns, Sharpe ratios, VaR and CVaR, and the diversification and concentration ratios of each portfolio strategy. We have analyzed the performance for the daily returns over a sample of 14 years, divided into two sub-samples of seven years each one, whose purpose is to test the robustness of the results in periods of high and low correlations between assets and with a market characterized by many bullish and bearish trends.

We have found that the Sharpe ratio of the mean-variance (MV) and the minimum variance (GMV) strategies are higher compared to the naive strategy of 1/N and the Ibex 35 index, in the 2001-2014 period. All models, achieved a Sharpe ratio greater than the Ibex 35 index during the 2001-2014 period, although only nine strategies are statistically different.

Regarding the total return for the 2001-2014 period, the MinCDaR95 portfolio is found to deliver higher returns, followed for the mean-variance, the Minimax optimization based on risk/return ratio, the low beta and the minimum variance strategies. All these obtained returns five times greater than those derived from the naive strategy of 1/N.

The performance of the naive strategy of 1/N is found not to be much different from other strategies in the 2001-2007 period, although its surpassed by five models, except

in concentration ratio, among which are the mean-variance (M-V) and the minimum variance (GMV) portfolios.

We observed that the 1/N strategy performance is worse than the Ibex 35 index in the 2008-2014 period. It is from 2008 when we detected divergences between the naive strategy of 1/N and the other strategies. Our findings suggest that the 1/N portfolio seems to show the worst performance during the Global Financial Crisis and the European Sovereign Debt Crisis (this is, a time period characterized by a higher correlation between financial assets and downtrends in the markets). Furthermore, except the O-Minimax portfolio, other strategies are found to outperform the naive strategy of 1/N in a lower VaR and CVaR, and a higher diversification ratio. We found that the 1/N portfolio has a lower degree of concentration, although it is to be expected since it includes all the assets that make up the Ibex 35 index. A large number of strategies have been found to produce a better performance than the Ibex 35 index and the naive strategy of 1/N. We have shown that most of strategies outperform both the Ibex35 index and the 1/N strateg, various portfolio strategies achieving higher return, greater Sharpe ratio, greater diversification ratio and lower VaR and CVaR than those associated with the naive strategy of 1/N and the Ibex 35 index.

In addition, our empirical results indicate that there are several strategies that do not depend on the expected assets return to assign weights (such as the GMV, ERC, MDP and MTD strategies) that are also able to overcome the naive strategy of 1/N. Nevertheless, the Markowitz mean-variance portfolio with shortselling constraint is found to be the only strategy that achieves a Sharpe ratio statistically different to the Ibex 35 in the two time periods analyzed (2001-2007 and 2008-2014). In view of the encouraging results of this paper work, we suggest that the mean-variance, minimum-variance and conditional draw-down at risk (95%) portfolios could be used, at least as a first reference, when analyzing the behavior of the main Spanish stock market.

All in all, the results of our analysis are not consistent with those presented in DeMiguel *et al.* (2009) and Allen *et al.* (2014A); although these are in line with those of Kirby and Ostdiek (2012) and Allen *et al.* (2014b) for the hedge fund indices. Thus, although as in all empirical works the results obtained have to be taken with some degree of caution (since they are referred to a particular index over a certain time period), our findings lead us to infer that the naive strategy of 1/N can provide a good results during some episodes, being always exceeded by several portfolio optimization models.

## Appendix A

In this appendix, we include two tables: Table A1 describes the 15 main statistical of the models used well as the Ibex 35 index. Table A2 offers the assets name that we have considered for the portfolio construction.

**Table A1**

**Summary of the main statistical, 2001-2014, 2001-2007 and 2008-2014 period.**

Portfolios	Time Period	Min	1st quartile	Median	Mean	3st quartile	Max
Ibex 35	2001-2014	-9.1408	-0.7367	0.0694	0.0151	0.7663	14.4349
	2001-2007	-5.8171	-0.6104	0.0866	0.0371	0.6838	5.9599
	2008-2014	-9.1408	-0.8937	0.0320	-0.0059	0.8756	14.4349
1/N	2001-2014	-8.1174	-0.6218	0.0644	0.0194	0.7184	10.8051
	2001-2007	-5.3981	-0.4502	0.0900	0.0543	0.6279	4.0264
	2008-2014	-8.1174	-0.8157	0.0389	-0.0149	0.8254	10.8051
M-V	2001-2014	-7.1356	-0.5125	0.0815	0.0433	0.6068	10.5063
	2001-2007	-3.6690	-0.3683	0.0802	0.0577	0.5077	5.9757
	2008-2014	-7.1356	-0.6975	0.0858	0.0291	0.7327	10.5063
GMV	2001-2014	-6.9594	-0.4884	0.0638	0.0402	0.5883	10.0633
	2001-2007	-3.9029	-0.3608	0.0869	0.0601	0.4840	6.1715
	2008-2014	-6.9594	-0.6285	0.0485	0.0206	0.7007	10.0633
MDP	2001-2014	-7.4434	-0.4782	0.0709	0.0346	0.6107	10.8855
	2001-2007	-4.1242	-0.3760	0.0780	0.0498	0.5113	6.6558
	2008-2014	-7.4434	-0.6320	0.0683	0.0197	0.7065	10.8855
ERC	2001-2014	-7.7076	-0.5593	0.0663	0.0241	0.6684	10.5324
	2001-2007	-4.9052	-0.3944	0.0913	0.0545	0.5822	3.7957
	2008-2014	-7.7076	-0.7731	0.0366	-0.0059	0.7634	10.5324
MTD	2001-2014	-6.7086	-0.5082	0.0715	0.0282	0.6168	10.9713
	2001-2007	-4.5205	-0.39020	0.0675	0.0509	0.5473	5.7833
	2008-2014	-6.7086	-0.6882	0.0798	0.0058	0.7177	10.9713
CVaR	2001-2014	-7.1332	-0.5122	0.0424	0.0323	0.6066	11.5357
	2001-2007	-4.4808	-0.3877	0.0490	0.0528	0.5217	6.6697
	2008-2014	-7.1332	-0.7053	0.0235	0.0121	0.7347	11.5357
MaxDD	2001-2014	-11.5167	-0.6781	0.0263	0.0373	0.7447	13.7449
	2001-2007	-6.0443	-0.5381	0.0279	0.0515	0.6501	5.7198
	2008-2014	-11.5167	-0.8617	0.0175	0.0233	0.8961	13.7449
AvDD	2001-2014	-13.0517	-0.8659	0.0007	0.0489	0.9324	16.7182
	2001-2007	-9.2679	-0.7592	0.0284	0.0496	0.8220	12.6075
	2008-2014	-13.0517	-0.9841	0.0000	0.0483	1.07235	16.7182
CDaR95	2001-2014	-10.7980	-0.7045	0.0225	0.0387	0.7942	12.9975
	2001-2007	-6.1414	-0.5546	0.0485	0.0542	0.6864	5.2095
	2008-2014	-10.7980	-0.9299	0.0000	0.0235	0.9433	12.9975
MinCDaR95	2001-2014	-7.3623	-0.5445	0.0339	0.0455	0.6411	10.5430
	2001-2007	-5.0936	-0.4068	0.0368	0.0564	0.5224	10.5430
	2008-2014	-7.3623	-0.6770	0.0269	0.0348	0.7544	10.3252
R-Minimax	2001-2014	-7.0666	-0.5786	0.0268	0.0278	0.6569	11.2922
	2001-2007	-4.3382	-0.4408	0.0303	0.0508	0.5370	4.35426
	2008-2014	-7.0666	0.754	0.0233	0.0050	0.7896	11.2922
O-Minimax	2001-2014	-8.8971	-0.6851	0.0576	0.0441	0.7309	11.3846
	2001-2007	-6.4801	-0.5924	0.0597	0.0361	0.6588	7.8800
	2008-2014	-8.8971	-0.7579	0.0368	0.0520	0.8326	11.3846
Clayton (MTD)	2001-2014	-7.2840	-0.5064	0.0764	0.0360	0.6483	10.9520
	2001-2007	-4.5469	-0.3797	0.0895	0.0598	0.5767	4.9972
	2008-2014	7.2840	-0.6836	0.0567	0.0124	0.7741	10.9520
Beta	2001-2014	-7.0623	-0.4720	0.0619	0.0405	0.6003	11.2040
	2001-2007	-4.5521	-0.3687	0.0671	0.0589	0.5053	6.3157
	2008-2014	-7.0623	-0.6338	0.0531	0.0224	0.6727	11.2040

**Table A2****Assets for the portfolio construction by time period.**

<b>Time period</b>	<b>N° of risky assets</b>	<b>Asset names</b>
03/01/2000 28/12/2001	21	Abertis, Acciona, ACS, Banco Popular, Bankinter, BBVA, Endesa, FCC, Gas Natural, Iberdrola, Mapfre, OHL, Repsol, Sacyr, Santander, Telefónica, Ferrovial, Indra, Ebro Foods, Sogecable, Altadis.
02/01/2001 30/12/2002	23	Abertis, Acciona, ACS, Banco Popular, Bankinter, BBVA, Endesa, FCC, Gas Natural, Iberdrola, Mapfre, OHL, Repsol, Sacyr, Santander, Telefónica, Ferrovial, Indra, Ebro Foods, Sogecable, Altadis, Zeltia, REE.
02/01/2002 30/12/2003	25	Abertis, Acciona, ACS, Banco Popular, Bankinter, BBVA, Endesa, FCC, Gas Natural, Iberdrola, Mapfre, OHL, Repsol, Sacyr, Santander, Telefónica, Ferrovial, Indra, Ebro Foods, Sogecable, Altadis, Zeltia, REE, Gamesa, Inditex.
02/01/2003 30/12/2004	26	Abertis, Acciona, ACS, Banco Popular, Bankinter, BBVA, Endesa, FCC, Gas Natural, Iberdrola, Mapfre, OHL, Repsol, Sacyr, Santander, Telefónica, Ferrovial, Indra, Ebro Foods, Sogecable, Altadis, Zeltia, REE, Gamesa, Inditex, Enagas.
02/01/2004 30/12/2005	29	Abertis, Acciona, ACS, Banco Popular, Bankinter, BBVA, Endesa, FCC, Gas Natural, Iberdrola, Mapfre, OHL, Repsol, Sacyr, Santander, Telefónica, Ferrovial, Indra, Ebro Foods, Sogecable, Altadis, Zeltia, REE, Gamesa, Inditex, Enagas, Banco Sabadell, Banesto, Prisa.
03/01/2005 29/12/2006	29	Abertis, Acciona, ACS, Banco Popular, Bankinter, BBVA, Endesa, FCC, Gas Natural, Iberdrola, Mapfre, OHL, Repsol, Sacyr, Santander, Telefónica, Ferrovial, Indra, Ebro Foods, Sogecable, Altadis, REE, Gamesa, Inditex, Enagas, Banco Sabadell, Banesto, Prisa, Atresmedia, Mediaset.
02/01/2006 28/12/2007	30	Abertis, Acciona, ACS, Banco Popular, Bankinter, BBVA, Endesa, FCC, Gas Natural, Iberdrola, Mapfre, OHL, Repsol, Sacyr, Santander, Telefónica, Ferrovial, Indra, Ebro Foods, Sogecable, Altadis, REE, Gamesa, Inditex, Enagas, Banco Sabadell, Banesto, Prisa, Atresmedia, Mediaset, Banesto.
02/01/2007 30/12/2008	29	Abertis, Acciona, ACS, Banco Popular, Bankinter, BBVA, Endesa, FCC, Gas Natural, Iberdrola, Mapfre, OHL, Repsol, Sacyr, Santander, Telefónica, Ferrovial, Indra, Ebro Foods, REE, Gamesa, Inditex, Enagas, Banco Sabadell, Mediaset, Banesto, BME, Grifols, Abengoa.
02/01/2008 30/12/2009	30	Abertis, Acciona, ACS, Banco Popular, Bankinter, BBVA, Endesa, FCC, Gas Natural, Iberdrola, Mapfre, OHL, Repsol, Sacyr, Santander, Telefónica, Ferrovial, Indra, Ebro Foods, REE, Gamesa, Inditex, Enagas, Banco Sabadell, Mediaset, Banesto, BME, Grifols, Abengoa, Técnicas Reunidas.
02/01/2009 30/12/2010	31	Abertis, Acciona, ACS, Banco Popular, Bankinter, BBVA, Endesa, FCC, Gas Natural, Iberdrola, Mapfre, OHL, Repsol, Sacyr, Santander, Telefónica, Ferrovial, Indra, Ebro Foods, REE, Gamesa, Inditex, Enagas, Banco Sabadell, Mediaset, Banesto, BME, Grifols, Abengoa, Técnicas Reunidas, Arcelormittal.
04/01/2010 30/12/2011	31	Abertis, Acciona, ACS, Banco Popular, Bankinter, BBVA, Endesa, FCC, Gas Natural, Iberdrola, Mapfre, OHL, Repsol, Sacyr, Santander, Telefónica, Ferrovial, Indra, Ebro Foods, REE, Gamesa, Inditex, Enagas, Banco Sabadell, Mediaset, Banesto, BME, Grifols, Abengoa, Técnicas Reunidas, Arcelormittal.
03/01/2011 31/12/2012	32	Abertis, Acciona, ACS, Banco Popular, Bankinter, BBVA, Endesa, FCC, Gas Natural, Iberdrola, Mapfre, OHL, Repsol, Sacyr, Santander, Telefónica, Ferrovial, Indra, Ebro Foods, REE, Gamesa, Inditex, Enagas, Banco Sabadell, Mediaset, BME, Grifols, Abengoa, Técnicas Reunidas, Arcelormittal, Amadeus, Caixabank.
02/01/2012 31/12/2013	33	Abertis, Acciona, ACS, Banco Popular, Bankinter, BBVA, Endesa, FCC, Gas Natural, Iberdrola, Mapfre, OHL, Repsol, Sacyr, Santander, Telefónica, Ferrovial, Indra, REE, Gamesa, Inditex, Enagas, Banco Sabadell, Mediaset, BME, Grifols, Técnicas Reunidas, Arcelormittal, Amadeus, Caixabank, DIA, Bankia, IAG.
02/01/2013 31/12/2014	35	Abertis, Acciona, ACS, Banco Popular, Bankinter, BBVA, FCC, Gas Natural, Iberdrola, Mapfre, OHL, Repsol, Sacyr, Santander, Telefónica, Ferrovial, Indra, REE, Gamesa, Inditex, Enagas, Banco Sabadell, Mediaset, BME, Grifols, Técnicas Reunidas, Arcelormittal, Amadeus, Caixabank, DIA, Bankia, IAG, Viscofan, Jazztel, Ebro Foods.

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